Eur. Phys. J. A 4, 83–90 (1999)

THE EUROPEAN
PHYSICAL JOURNAL A

A unified scheme for flavoured mesons and baryons

P.C. Vinodkumar[1,3], J.N. Pandya[1], V.M. Bannur[2], S.B. Khadkikar[3]

[1] Dept. of Physics, Sardar Patel University, Vallabh Vidyanagar, Gujarat, India - 388 120
[2] Institute for Plasma Research, Bhat, Gandhinagar, Gujarat, India - 382 428
[3] IUCAA, P. O. Box No. 4, Ganeshkhind, Pune, India - 411 007

Received: 26 February 1998 / Revised version: 1 September 1998
Communicated by F. Lenz

Abstract. The masses of mesons and baryons with various flavour combinations for $q\bar{q}$, $q\bar{Q}$, $Q\bar{Q}$, qqq, qqQ, qQQ, QQQ etc. are computed using a confinement scheme based on harmonic approximation with Lorentz scalar plus vector character. The residual two body coulombic interactions and the spin-hyperfine interactions of the confined one gluon exchange effects are perturbatively calculated and added to the confinement energy to get the mass of the hadron. With all the parameters fixed to get ground state masses of hadrons containing like flavour combinations, a parameter free prediction of the leptonic decaywidths of vector mesons and their sizes are being made. Our results on the baryonic and mesonic masses with open flavours and the predictions on the leptonic decay widths are in good agreement with the respective experimental values.

PACS. 12.10.Dm Unified theories and models of strong and electroweak interacting

1 Introduction

In recent years, there has been a renewed interest in the spectroscopy of hadrons due to a number of experimental facilities (CLEO, DELPHI etc.) which have provided and are expected to provide more accurate and new informations about hadrons from low flavours to heavy flavours [1-3]. Among the many theoretical attempts and approaches to explain the hadron properties based on its quark structure a very few were successful in predicting the hadronic properties starting from low flavour to heavy flavours. Apart from some recent algebraic attempts which lack the probable link with the theoretical approximations of qcd for the confinement of coloured quarks [4], most of the properties of hadrons are reported in a potential frame work. The non-relativistic potential models with linear plus coulomb, power potential [5, 6] etc. were successful at the heavy flavour sectors, while the nonrelativistic harmonic oscillator potential [7] as well as the Bethe-Salpeter approach under harmonic confinement [8] were successful at low flavour sectors. Though some of these successful model could reproduce the masses of the low-lying hadrons, their predictions for the leptonic decay widths of vector mesons are not satisfactory with respect to the experimental values [9, 10]. Here we would like to mention one of our earlier work on effective theory of small fluctuations around a background classical solution of Yang-Mills theory [11], where we have been able to provide a physical basis for the heuristic nature of the phenomenological confinement schemes with Lorentz scalar plus Lorentz vector character such as the relativistic harmonic confinement model (RHM) for quarks and the current confinement model (CCM) for gluons, in the form of condensate of the background colour fields. These models have already been employed for the study of the magnetic moments of baryons, light flavour low-lying masses of hadrons and for the study of glueballs [12, 13]. They were also instrumental for the derivation of the residual effective (confined) one gluon exchange potential (COGEP) through Breit formalism under the Born approximation [14], and have been successfully employed for the study of nucleon-nucleon scattering phase shifts in quarks cluster calculations [15, 16]. In this context, one would like to employ this scheme with the desired properties of qcd, for the study of hadrons from the low flavours to heavy flavours. Hence the present study is an attempt to understand the spectroscopy of the hadrons, in particular, the masses and leptonic decay widths of the vector mesons and the masses of baryons with different flavour compositions containing light as well as heavy quarks, in the frame work of RHM with the residual confined one gluon exchange potential (COGEP) between the confined quarks.

The details of the present study are presented as follows. In Sect. 2, the theoretical formulation leading to the computations of the hadronic masses with different quark/antiquark combinations are discussed. Apart from the confinement energy (the dynamical internal mass) of quarks in the mean field potential provided by the RHM, the residual two body coulomb interactions among the constituent confined quarks are computed to get the spin averaged centre of weight (COW) masses of mesons and baryons. The spin hyperfine interaction given by the residual effective (confined) one gluon exchange potential (COGEP) employed in our earlier studies [14, 15, 16] is used here to get the mass difference of the pseudoscalar-vector mesons as well as the spin 1/2 - spin 3/2 baryons. With all the parameters fixed to yield the mesonic masses of the like flavour combinations, we predict the masses of the open flavour mesons and baryons in the $\ell = 0$ states. We also compute the leptonic decay widths of the vector mesons and their sizes with no other additional free parameters. The details are presented in Sect. 3 of this paper. Finally we conclude in Sect. 4 by summerising the main features of the present study while comparing our results with other theoretical models.

2 Theoretical formulation of the unified scheme for hadronic masses

The properties of hadrons are calculated in a potential frame work using a mean field confinement potential with residual colour electrostatic and chromomagnetic interactions added perturbatively. Accordingly, the mass of a hadron in an energy eigen state N and spin state J containing p number of different quarks or quark-antiquark combinations in general can be calculated using the formula,

$$M_N^J(q_1, q_2, ...q_p) = \sum_{i=1}^{p} \epsilon_N(q_i, p)_{conf} + \sum_{i<j=1}^{p} \epsilon_N(q_i, q_j)_{coul} + \sum_{i<j=1}^{p} \epsilon_N^J(q_i, q_j)_{S.D.} \quad (1)$$

Here we have $p = 2$ for mesons and $p = 3$ for baryons. The first term in the above mass expression corresponds to the total confinement energy (total dynamical inertial mass) of the constituent quarks, which are computed non-perturbatively using the scheme described by RHM. The second term which is the residual coulomb energy among the confined constituent quarks, corresponds to the residual colour electrostatic interactions. The third term corresponds to the spin-dependent interaction energy (spin-hyperfine, spin-orbit and tensor etc.) among the confined constituent quarks. These energies are calculated perturbatively and are added to the confinement energy. For total orbital angular momentum $\ell = 0$ states the spin-orbit and tensor terms do not contribute. Presently we restrict our calculations for the S-state hadrons and retain terms upto the spin-hyperfine. The spin-averaged (centre of weight) masses of the hadrons are the one without the spin dependent term of (1) and is reperesented by $\overline{M}_N(q_1, q_2, ...q_p)$.

We briefly review the relativistic harmonic confinement mean field potential (RHM) applied here in computing the confined energy $\epsilon_N(q_i, p)_{conf}$ of the participating quark (q_i) in the eigenstate N of a p-quarks system. In this scheme, the quarks inside a p-quarks system (hadron) are under the action of a Lorentz scalar plus a Lorentz vector mean field potential of the form [12],

$$V_{conf} = \frac{1}{2}(1 + \gamma_o)A^2 r^2 \quad (2)$$

where A is the confinement mean field parameter, γ_o is the Dirac matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and r is the mean distance of the quark from the p-quark centre. The confined single particle state of the quarks under this mean field potential is described by the Dirac equation,

$$[i\gamma^\mu \partial_\mu - M_q - V(r)]\, \psi_q(r) = 0 \quad (3)$$

where the quark wave functions $\psi_q(r)$ expressed in bispinor form $[\chi_q,\, \phi_q]$ satisfy the coupled equation,

$$\left[E - M_q - A^2 r^2\right] \chi_q = -i\sigma. \bigtriangledown \phi_q \quad (4a)$$

and

$$[E + M_q]\phi_q = -i\sigma. \bigtriangledown \chi_q \quad (4b)$$

Here M_q is the quark mass parameter and E is the eigenvalue of the single particle Dirac equation. Using a transformation as shown below, the lower component of the bispinor can be eliminated as [15],

$$U\psi_q = \frac{1}{\left[1 + \frac{p^2}{(E+M_q)^2}\right]} \begin{bmatrix} 1 & \frac{\sigma.\mathbf{p}}{|E+M_q|} \\ \frac{-\sigma.\mathbf{p}}{|E+M_q|} & 1 \end{bmatrix} \times \begin{pmatrix} \chi_q \\ \phi_q \end{pmatrix} = \begin{pmatrix} \chi_q \\ 0 \end{pmatrix} \quad (5)$$

such that $\langle\psi_q|\psi_q\rangle = \langle\chi_q|\chi_q\rangle$ and are normalised. The upper component χ_q now satisfies

$$\left[-\bigtriangledown^2 + A^2 r^2 (E + M_q)\right] \chi_q = (E^2 - M_q^2)\chi_q \quad (6)$$

which resembles the three dimensional harmonic oscillator equation with energy dependent size parameter,

$$\Omega_N(q) = A(E_N + M_q)^{1/2} \quad (7)$$

and the radial solution of (6) is given by,

$$R_{nl}(r) = \left[\frac{\Omega_N^{3/2}}{2\pi} \frac{n!}{\Gamma(n + l + \frac{3}{2})}\right]^{1/2} \times (\Omega_N^{1/2} r)^l exp\left(\frac{-\Omega_N r^2}{2}\right) L_n^{l+\frac{1}{2}}(\Omega_N r^2) \quad (8)$$

Here the oscillator eigenvalue $2N+3$ occurs in the expression for E_N^2 instead of E_N of a nonrelativistic oscillator potential. From the eigenvalue expression for (6), the single particle energy of the quark is given by

$$E_N = \pm\sqrt{M_q^2 + (2N+3)\Omega_N(q)} \tag{9}$$

Following Dirac, the negative energy states are interpreted as antiparticles. Further in the limit, momentum $p \to 0$, the eigenvalue becomes proportional to the distance r which is consistent with the lattice result of the linear behaviour of confinement potential. The energy quantum number $N = 2n + \ell = 0, 1, 2....$, where n is the radial quantum number with ℓ as the angular momentum quantum number. Though the RHM mean field potential described by (2) has r^2 like confinement, the confinement energy goes as square root of the respective eigenvalue equation with energy dependent size parameter. This particular attribute of the mean field relativistic harmonic confinement model (RHM) makes it different from other potential models. And it has motivated us to employ the scheme for the study of heavy flavour mesons as well as baryons. Preliminary calculations of the hadronic masses with different flavour content [17] has encouraged us to do the detailed computations incorporating the residual effective (confined) one gluon exchange effects.

Thus we have the single particle energy expression for quarks and antiquarks as a function of the confinement parameter A and the mass parameter M_q for each eigenstate. Here we treat quarks and antiquarks on equal footing with their energy represented by $|E_N|$. Moreover, the transformations given by (5) enable us to treat the system nonrelativistically. Energies of the mesons and baryons ($q\bar{q}$, $Q\bar{Q}$, qqq, qQQ, qqQ etc.) are constructed from these single particle energies by accounting for the spurious motion of the centre. We adopt a scheme similar to the one which was used for the predictions of the low lying baryonic masses by keeping the 'center of mass' at the lowest eigenstate [12]. Then the intrinsic energies of the participating quarks are obtained by subtracting their contribution to the centre of mass from their single particle energy. Accordingly the intrinsic energies of the quarks in a p quark system (for mesons, $p = 2$ and for baryons $p = 3$) are given by

$$\epsilon_N(q,p)_{conf} = \sqrt{(2N+3)\Omega_N(q) + M_q^2 - \frac{3M_q}{\sum_{i=1}^{p} M_q(i)}\Omega_0(q)} \tag{10}$$

The confinement energies of the hadrons can then be obtained by adding the respective intrinsic energies of the chosen flavour combinations from (10).

The residual confined one gluon exchange interaction, particularly the static coulombic interaction between the confined quarks is treated here perturbatively in the confinement basis and added according to (1) to get the spin independent centre of weight mass of the hadron as,

$$\overline{M}_N(q_1q_2.....) = \sum_{i=1}^{p} \epsilon_N(q_i,p)_{conf} + \sum_{i<j=1}^{p} \epsilon_N(q_iq_j)_{coul} \tag{11}$$

The coulombic part of the energy is computed using the residual coulomb potential given by [18],

$$V_{coul}(q_iq_j) = \frac{\alpha_s^{eff}(\mu)}{r} \tag{12}$$

where $\alpha_s^{eff}(\mu)$ is related to the perturbative $\alpha_s(\mu)$ as,

$$\alpha_s^{eff}(\mu) = \frac{\alpha_s(\mu)}{\omega_n} \tag{13}$$

where ω_n is the colour dielectric "coefficient" [18]. Such medium effects reflect the noncoulombic form of the static colour coulomb force derived recently using charmonium states [19]. In the present calculations, we found ω_n to be state (energy) dependent so as to get consistent coulombic contribution to the excited states of the hadrons. Using the centre of weight masses of $c\bar{c}$ states, we determine the values of ω_n. The colour dielectric coefficient deduced from the $c\bar{c}$ spectrum reflects the nature expected for the gross property of QCD [18]. It has been found that the effective colour dielectric coefficient can be parametrised as,

$$\omega_n = k\sqrt{\frac{\alpha_s^3(\mu)}{\alpha_m(\mu)}}\left\{2^{(n-1)/2}[2^{n+1} - (-1)^n]\right\}^{-1} \tag{14}$$

where $\alpha_m(\mu)$ is a flavour dependent deimensionless model parameter defined as

$$\alpha_m(\mu_q) = \left[\frac{A^{2/3}}{\mu_q}\right] \tag{15}$$

It is a measure of the confinement strength through the nonperturbative contributions to the confinement scale at the respective threshold energy of the given flavour production. The strong running coupling constant $\alpha_s(\mu)$ is computed using

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_o)}{1 + \frac{33-2n_f}{12\pi}\alpha_s(\mu_o) ln\left(\frac{\mu}{\mu_o}\right)^2} \tag{16}$$

here $\alpha_s(\mu_o)$ is the strong coupling constant at the threshold production of mesons in the (uds) sector with a threshold energy around 1 GeV. In the present calculation we have taken the value of $\alpha_s(\mu_o) = 0.7$ and the $\alpha_s(\mu)$ for heavy systems are calculated using (16).

It can be shown that the $\alpha_m(\mu_q)$ defined in (15) vary with $\alpha_s(\mu)$ as,

$$\alpha_m(\mu_q) = \frac{A^{2/3}}{\Lambda_{QCD}} exp\left[\frac{-6\pi}{(33-2n_f)\alpha_s(\mu)}\right] \tag{17}$$

It has the right behaviour with $\alpha_s(\mu)$ at the asymptotic limit as $\alpha_s(\mu) \to 0$, then $\alpha_m(\mu_q) \to 0$ exponentially. Hence

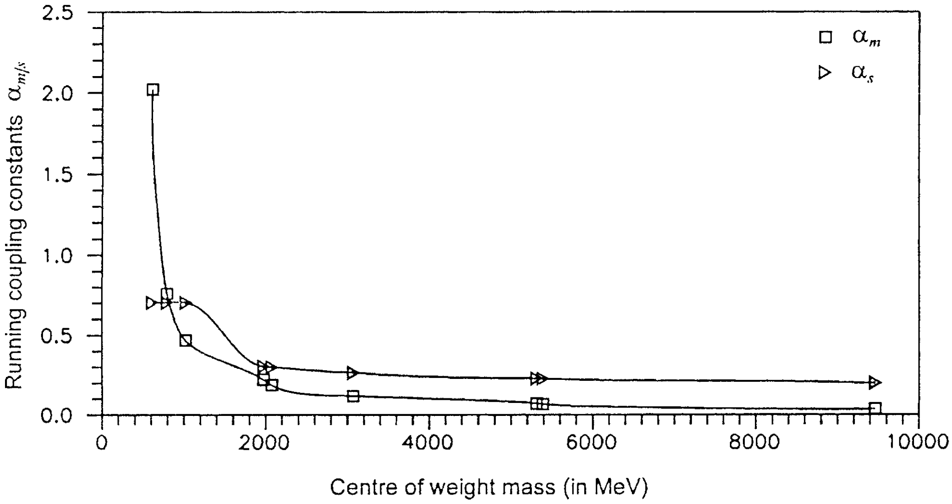

Fig. 1. The running coupling constants α_m and α_s as a function of centre of weight mass

Table 1. a. Parameters used in the calculations of the spin-averaged mass of the hadrons

Confinement mean field paramater, $A = 2166$ (MeV)$^{3/2}$ as in Ref. [12,14,15]
$M_u = M_d = 82.8$ MeV; $M_s = 357.5$ MeV; $M_c = 1428$ MeV; $M_b = 4636.6$ MeV; $M_t = 177$ GeV (Ref. [25])
The medium parameter, $k = 5.1943$ (for mesons) and 11.0252 (for baryons)
$C_o = 1.47109$ (for all hadrons)

Table 1. b. Parameters used for the spin-hyperfine calculations

C_{CCM} for baryons = 50.00 MeV
C_{CCM} for mesons = 56.6 MeV (*uds* sector);
99.82 MeV (charmed mesons);
142.5 MeV (beauty mesons)

coulombic strength vanishes as $\alpha_s(\mu) \to 0$. The effective radial coulomb energy $\epsilon_N(q_iq_j)_{coul}$ then contains an implicit flavour dependence through $\alpha_s(\mu)$ and $\alpha_m(\mu)$ and an explicit state dependence through the radial quantum number 'n'.

Now we make an ansatz to obtain the wave functions of the two quark systems which are constructed from the RHM single particle wave function using a simple additive rule for the two particle size parameter. It can be shown through the Moshinsky transformation [20] that the form of the wave function for a two/three particle systems are qualitatively similar to the single particle wave functions in the oscillator model but for their size parameter. Thus using RHM we construct the wave functions for the two quark systems by retaining the nature of single particle wave function but with a two particle size parameter $\Omega_N(q_iq_j)$ instead of $\Omega_N(q)$ appearing in (8). We construct $\Omega_N(q_iq_j)$ for ℓ=0 states as,

$$\Omega_N(q_iq_j) = C_N(q_i)\Omega_N(q_i) + C_N(q_j)\Omega_N(q_j) \tag{18}$$

here $C_N(q)$ are related to the Moshinsky transformation coefficients [20]. For simplicity, we approximate these coefficients and parameterise in terms of the ratio of two dimensionless parameters α_m and α_s, so as to make it convenient to generalise for hadronic systems containing multiquarks (i.e. $p \geq 3$). Thus, for p - quark system we may define a single centre $\Omega_N(q_1, ...q_p)$ as

$$\Omega_N(q_1, ...q_p) = \frac{C_o}{p^n} \sum_{i=1}^{p} \frac{\alpha_s(\mu_i)}{\alpha_m(\mu_{q_i})} \Omega_N(q_i) \tag{19}$$

where n is the radial quantum number given by $(N - l)/2$, C_o is a constant weight factor. Now coulomb energy is computed perturbatively using the confinement basis with two particle size parameter defined above for different states as,

$$\epsilon_n(q_iq_j)_{coul} = \langle N|V_{coul}|N\rangle \tag{20}$$

From (11) with the help of (10) and (20), we compute the centre of weight mass of the hadrons from the light flavour sector to the heavy flavour sector. The confinement parameter A has been taken as the same value used for the study of the low lying baryons and their magnetic moments [12]. Thus in the present study, the free parameters are fixed to yield the ground state masses of the mesons of the like flavour combinations using contour plots for their ground state energies. The parameters used here for the computation of the centre of weight masses of the hadrons are listed in Table 1a. The dimensionless running parameter $\alpha_m(\mu_q)$ and $\alpha_s(\mu)$ are plotted in Fig. 1 as a function of the centre of weight mass of the respective mesons. The behaviour of the plot clearly indicates the importance of nonperturbative calculations around 1 GeV and the validity of perturbative calculations at higher threshold energies with a critical α_s. The behaviour of our plot is found to be similar to that of $\alpha_s(Q^2)$ with a parametrised form for $\alpha_s(Q^2)$ at $Q^2 \to 0$ given by Capstic and Isgur in their study of baryons [9]. The computed centre of weight masses for the various hadrons in open flavour are listed in Table 2 along with the corresponding values deduced from the experimental masses of the

Table 2. Centre of weight mass of mesons and baryons having different flavour contents

Mesons / Baryons	Masses in MeV	
$(u=d=q)$	Theoretical	Experimental
$q\bar{q}$	<u>612</u>	612
$q\bar{s}$	817	793
$q\bar{c}$	1912	1974
$q\bar{b}$	5222	5312
$s\bar{s}$	<u>1020</u>	1020
$s\bar{c}$	2085	2074
$s\bar{b}$	5382	5392
$c\bar{c}$	<u>3068</u>	3068
$c\bar{b}$	6300	
$b\bar{b}$	9452	
$t\bar{t}$	354310	–
qqq	<u>1085</u>	1085
qqs	1287	1287
qqc	2358	2369
qqb	5606	–
qss	1490	1426
qsc	2545	–
qsb	5793	–
qcc	3555	–
qcb	6783	–
qbb	9979	–
sss	1689	1672
ssc	2732	2704
ssb	5980	–
scc	3741	–
scb	6969	–
sbb	10165	–
ccc	4739	–
ccb	7955	–
cbb	11147	–
bbb	14329	–

hadrons using

$$M_{hh'} = M_h - \frac{|SF_{h'}|(M_h - M_{h'})}{|SF_h| + |SF_{h'}|} \tag{21}$$

Here SF_h's correspond to the spin factor for the hadron h and M_h's are the experimental masses of the hadrons h and h' with similar flavour content.

From the centre of weight masses, the pseudoscalar as well as the vector mesonic masses and the baryon octet ($J = 1/2$) and decuplet ($J = 3/2$) masses are obtained by incorporating the residual two body chromomagnetic interaction through the spin-dependent term of (1) as,

$$\epsilon_N^J(q_i q_j)_{S.D.} = \langle NJ|V_{\sigma_1.\sigma_2}|NJ\rangle \tag{22}$$

Here, we consider the spin-hyperfine interaction of the residual (effective) confined one gluon exchange potential (COGEP) as,

$$V_{\sigma_i.\sigma_j} = \frac{\alpha_s(\mu)N_i^2N_j^2}{4}\frac{\lambda_i.\lambda_j}{[E_i+m_i][E_j+m_j]} \times [4\pi\delta^3(r) - C^4_{CCM}r^2D_1(r_{ij})]\left(-\frac{2}{3}\sigma_i.\sigma_j\right) \tag{23}$$

where $D_1(r_{ij})$ is the confined gluon propagator and $N_{i/j}$ are the RHM normalisation constants given as [14, 15],

$$D_1(r_{ij}) = \frac{\Gamma(3/4)}{4\pi^{3/2}}C_{CCM}(C_{CCM}r_{ij})^{-3/2}W_{0,-1/4}(C^2_{CCM}r^2_{ij}) \tag{24a}$$

and

$$N_{i/j} = \sqrt{2(E_{i/j}+M_{i/j})/(3E_{i/j}+M_{i/j})} \tag{24b}$$

Where, C_{CCM} corresponds to the confinement strength of the gluons, r_{ij} is the interquark distance and $W_{\lambda,k}$ represents the Whittaker function. In the present calculations, we treat the parameter C_{CCM} as the hyperfine parameter, which is to be fitted to get the ground state energy difference of the hadrons with same flavour content but with different spins. It is found that the single hyperfine parameter (C_{CCM}) obtained from the $N-\Delta$ masses could predict the masses of baryons from lower to heavier flavour combinations. Where as for mesons, the hyperfine parameter varies as per their centre of weights. It may be attributed due to the contributions like $q\bar{q}$ annihilation into hard gluons. Hence the hyperfine parameter is different for different threshold energies of the various flavour meson productions while for baryons such contributions are negligible and expected to have a single hyperfine parameter for all flavour sectors [21,22]. We obtain CCM parameter separately for mesons with lower flavour mesons, $c\bar{c}$ and $b\bar{b}$ systems so as to get the ground state masses of the respective spin 0 (pseudoscalar) and spin 1 (vector) mesons and are listed in Table 1*b*. The CCM parameters for open flavour mesons are computed as the average of CCM parameters weighted with the sum of the running coupling constants of the respective flavour content of the mesons. The computed ground state masses of the octet, decuplet baryons and vector, pseudoscalar mesons with different flavour combinations are tabulated in comparison with the experimental values [3] as well as with other theoretical model predictions [9,23] in Table 3*a* for baryons and in Table 3*b* for mesons [4,10] respectively.

3 Leptonic decay widths and charge radii of vector mesons

One of the crucial test for the success of any phenomenological model for the colour confinement is provided by its predictions of the leptonic decay widths of vector mesons. The contemporary confinement models are not satisfactory in yielding the absolute values of the leptonic decay widths [5, 24] using the Van - Royen - Weiskopf formula (V - W) [6]. In many cases, it overestimates although ratios of leptonic widths of excited states to that of the ground states are satisfactory. To deal with this problem, various

Table 3.a. Ground state masses of octet-decuplet baryons

Baryons	Masses in MeV in $1S$ state			
	Present	Expt.[3]	Ref. [9]	Ref. [23]
N	<u>939</u>	939	960	939
Δ	<u>1232</u>	1232	1230	1224
Σ	1181	1192	1190	1195
Σ^*	1393	1385	1370	1381
Ξ	1323	1315	1305	1316
Ξ^*	1557	1530	1505	1532
Ω	1689	1672	–	–
Σ_c	2310	2454	2440	2467
Σ_c^*	2407	–	2495	–
Ξ_c	2519	2470	–	–
Ξ_c^*	2572	2644†	–	–
Ω_c	2732	2704	–	–
Σ_b	5582	–	5795	–
Σ_b^*	5631	5641†(Λ_b)	5805	–

† Experimentally J^P values are not confirmed

Table 3.b. Ground state masses of pseudoscalar-vector mesons

Mesons	Masses in MeV in $1S$ state			
	Present	Expt.[3]	Ref. [10]	Ref. [4]
π	149	139	152	145
ρ	767	770	770	762
K	507	494	470	501
K^*	921	892	900	902
D	1758	1869	1880	1831
D^*	1964	2007	2040	1977
D_s	2000	1969	1980	2015
D_s^*	2113	2112†	2130	2148
η_c	<u>2979</u>	2979	2970	2997
J/ψ	<u>3097</u>	3097	3100	3089
B	5195	5279	5310	5117
B^*	5231	5325	5370	5176
B_s	5371	5369	5390	–
B_s^*	5386	–	5450	–
Υ	<u>9461</u>	9461	9460	9601
η_b	9424	–	9400	–
$t\bar{t}$	354310	–	–	–

† Experimentally J^P values are not confirmed

authors considered re-scaling of mesonic wave functions [5] and incorporated radiative corrections to the V - W formula. Even then the success were partial as some models were successful at the light flavour mesons [8] while the others were successful at the heavy flavour mesons [6].

In this context, it is desirable to compute the leptonic decay widths of vector mesons and their sizes from the single unified scheme similar to the one employed for the study of hadronic masses in this paper. The leptonic decay widths are computed using the V - W formula [5, 24] given by,

$$\Gamma_{v\to l^+l^-} = \frac{4\alpha^2 e_q^2}{M_v^2}|R_{nS}^{q_iq_j}(0)|^2 \tag{25}$$

here M_v is the mass of the vector meson in the n^{th} state, α is the electromagnetic fine structure constant (= 1/137), e_q is the charge content of the quark flavour composition of the meson and $R_{nS}^{q_iq_j}(0)$ is the radial wave function of the meson evaluated at the centre. Using the two particle size parameter $\Omega_{nS}(q_iq_j)$ calculated from (16) with p=2, we compute $|R_{nS}^{q_iq_j}(0)|^2$ for S-states (ℓ=0) as

$$|R_{nS}^{q_iq_j}(0)|^2 = \frac{\Omega_{nS}^{3/2}(q_iq_j)n!}{2\pi\Gamma(n+3/2)}\left[L_n^{1/2}(0)\right]^2 \tag{26}$$

and the scalar charge radii of the mesons in the given eigen state can be obtained as

$$\langle r_{nS}^2\rangle^{1/2} = \left[\int_0^\infty |R_{nS}^{q_iq_j}(r)|^2 r^2 r^2 dr\right]^{1/2} \tag{27}$$

As all the parameters here are fixed to yield hadronic masses, the computed values of the decay widths as well as their radii are parameter free predictions from the scheme presented in this paper. The computed values of the leptonic decay widths and the scalar charge radii of the vector mesons in various $\ell = 0$ states are listed along with their computed masses and compared with the respective known experimental values in Table 4. The present values of the masses of various excited $\ell = 0$ states of vector mesons are compared with other successful theoretical models [4,10]. While most of the models failed to reproduce the masses and the absolute value of leptonic decay widths simultaneously under a unified framework, the model based on covariant soliton dynamics [24] predicted the masses of vector mesons, their leptonic decay widths and the charge radii rather accurately. Hence we compare our computed values with their results also in Table 4.

4 Summary and conclusions

In the present study based on a harmonic approximation to the confinement scheme with a phenomenological parameterisation for the residual colour electrostatic potential and using the residual one gluon exchange potential for spin-hyperfine interactions, we have been able to compute the masses of mesons and baryons of different flavour combinations with a single confinement strength. Unlike in the general case of a potential model, here we are able to get a good predictions of the masses of the hadrons in various flavour combinations. For the mass calculations we have included an effective residual coulombic potential perturbatively in the confinement basis. The masses thus computed with a similar quark composition yield the respective centre of weight mass for the multiquark systems. Our results of the centre of weight masses of mesons and baryons in Table 2 are in good agreement with the respective experimental values. The underlined masses of the like flavour combinations are used for fixing the mass parameter (M_q) of the respective flavours. The medium parameter (k) for the mesons is fixed to yield $c\bar{c}(1S)$ centre of weight mass and that for baryons is fixed to yield the

Table 4. Masses, leptonic decay widths and r.m.s. radii of vector mesons in comparison with experimental as well as theoretical results

System	Masses (in MeV)					Leptonic Decay Widths Γ_V (in keV)					$\langle r_{ns}^2\rangle^{1/2}$ in fm	
	Present	Ref.[10]	Ref.[4]	Ref.[24]	Expt.	Present	Ref.[24]	Ref.[5][a]	Ref.[5][b]	Expt.	Present	Ref. [24]
$t\bar{t}(1S)$	354310	–	–	–	–	0.367	–	–	–	–	0.010	–
$t\bar{t}(2S)$	355286	–	–	–	–	0.196	–	–	–	–	0.021	–
$\Upsilon(1S)$	9461	9460	9601	9460	9461	1.320	1.27	1.10	2.85	1.32 ± 0.05	0.186	0.28
$\Upsilon(2S)$	10023	10000	9977	10025	10023	0.628	0.45	0.50	1.07	0.52 ± 0.03	0.400	0.62
$\Upsilon(3S)$	10326	10350	10333	10345	10355	0.263	0.19	0.32	0.76	–	0.707	1.189
$\Upsilon(4S)$	10574	10630	10670	10560	10580	0.104	0.11	0.23	0.63	0.25 ± 0.03	1.165	1.529
$\Upsilon(5S)$	10753	10880	10990	10830	10860	0.0404	0.07	–	–	0.31 ± 0.07	1.850	1.429
$J/\psi(1S)$	3097	3100	3089	3050	3097	5.469	4.73	4.80	7.82	5.26 ± 0.37	0.386	0.385
$J/\psi(2S)$	3674	3680	3687	3671	3686	2.140	1.17	1.73	3.83	2.14 ± 0.21	0.823	1.195
$J/\psi(3S)$	4073	4100	4189	4057	4040	0.796	0.26	0.98	2.79	0.75 ± 0.15	1.443	1.443
$J/\psi(4S)$	4420	4450	4629	–	4415	0.288	0.08	0.51	2.19	0.47 ± 0.1	2.358	1.545
$\phi(1S)$	1020	1020	1054	1020	1020	1.380	–	1.21 (Ref.[8])		1.37 ± 0.05	0.800	–
$\phi(2S)$	1675	1690	1572	–	1680	0.286	–	–		–	1.634	–
$\phi(3S)$	1892	1900	1844	–	1850	0.085	–	–		–	2.784	–
$\rho(1S)$	767	770	762	775	770	6.510	–	5.71 (Ref.[8])		6.77 ± 0.32	0.958	–
$\rho(2S)$	1441	1450	1393	–	1450	1.375	–	–		–	1.852	–
$\omega(1S)$	767	780	762	775	783	0.723	–	0.65 (Ref.[8])		0.6 ± 0.02	0.958	–
$\omega(2S)$	1441	1460	1393	1450	1420	0.153	–	–		–	1.852	–

[a] Using logarithmic potential [b] Using modified coulomb potential

$N-\Delta$ centre of weight mass. The confining mean field parameter A used in the present study is the same as used for the study of the baryonic magnetic moments [12] as well as for the study of $N-N$-scattering phase shifts based on RHM and COGEP [14-16]. The same spin-spin interactions of the residual confined one gluon exchange potential (COGEP) has been employed here to compute the respective masses of the pseudoscalar and vector mesons as well as the decuplet (J=3/2) and octet (J=1/2) baryonic masses. The present results for the ground state masses of the baryons are in better agreement with the experimental values compared to other theoretical models (See Table 3a) based on relativistic potential calculations [9, 22]. Similarly our results of mesonic ground state masses with various flavour combinations are compared with some of the successful theoretical models such as a relativistic potential model for mesons [10] and with a theoretical model based on a systematic investigation of hadronic properties in terms of spectrum generating algebra $U(4)\otimes SU_s(2)\otimes SU_f(6)\otimes SU_c(3)$ for $q\bar{q}$ mesons [4]. Since the pseudoscalar partner of $\Upsilon(1S)$ is not yet experimentally known, our predictions of $\eta_b(1S)$ is comparable with the other model predictions [10,24]. It is quite evident from the Tables 3a and 3b that the present study could successfully predict many of the low-lying baryons as well as mesons from low flavour to the heavy flavour sectors. Our results on D and B-mesons and Σ_c baryons are not satisfactory though their hyperfine interactions are comparable.

The striking feature of the present study is that we have been able to do parameter free predictions of the leptonic decay widths of vector mesons upto few excited S-states and their sizes. Unlike many of the theoretical models, here we have been able to get consistent predictions of the masses, leptonic decay widths and the charge radii of the vector mesons. For $\Upsilon(b\bar{b})$ as well as for $J/\psi(c\bar{c})$ systems we have listed the properties upto $5S$ states while for the light flavour, $s\bar{s}$ and $u\bar{u}/d\bar{d}$'s, we have listed upto $3S$ and $2S$ only, though in principle we can get theoretical values for any excited states, but only very few excited states of these mesons are known experimentally [3]. Using the current experimental value of top quark mass [25] we have calculated the masses, leptonic decay widths as well as the sizes of $t\bar{t}$ meson upto $2S$ state eventhough the formation of $t\bar{t}$ bound state is very weak as t-quark decays faster than it could form a bound state. But there are theoretical model predictions of the properties of $t\bar{t}$ mesons [6] with the top quark mass much lower than the current value of $M_t = 177$ GeV [25]. Hence, we don't compare our results for $t\bar{t}$ mesons with other results.

From our results, it is clear that the present study predicts the masses and the leptonic widths (of low-lying states) of various mesons fairly well. However, the values of the leptonic widths of $b\bar{b}$ mesons beyond $3S$ shows some disagreement with the calculated values. While for the $c\bar{c}$ case, the hyperfine spectra as well as the decay widths are in close agreement with the respective experimental values. For ϕ, ω and ρ, the calculations reproduced the experimental results quite satisfactorily. Single C_{CCM} parameter for baryons and different C_{CCM} for the mesons suggest the importance of hard gluonic effects in the case of mesons which then may vary according to the threshold energy of the flavour production. For the charge radii of these mesons, our results are as per the expectation and are comparable to the values obtained in a covariant soliton model at the ground state [24]. The low lying baryonic masses from the uds sector to the b-quark sector obtained from the present study contain many predictions, eventhough many of the charmed and beauty baryons are yet to be established experimentally. We hope and look forward the confirmation of these heavy hadrons in future experiments. One of the important aspect of the present study is the description of the mesons and baryons with different flavour combinations in a single consistent unified framework. The leptonic decay widths of vector mesons upto a few excited states accomplishes this unification as we have no additional free parameters. We also obtained good results for the masses of open flavour mesons and baryons. We found that the octet-decuplet hyperfine splitting has common origin to the pseudoscalar-vector mesons but for different strength of the gluonic effects.

The present study has also enlightened us about the behaviour of an effective running coupling constant $\alpha_m(\mu_q)$ in the infrared region. The strong running coupling constant $(\alpha_s(\mu))$ computed using (16) largely corresponds to the perturbative interactions, while the model runnig coupling constant $\alpha_m(\mu_q)$ defined using the confinement strength and the masses of the quarks (15), corresponds to the nonperturbative interactions. From the plot of $\alpha_s(\mu)$ and $\alpha_m(\mu_q)$ against the centre of weight masses obtained in the present study (see Fig. 1) clearly demark the nonperturbative as well as perturbative domain with a critical running coupling constant $\alpha_c(\mu) \sim 0.7$, which is close to a similar value of 0.6 obtained through quark-quark correlator study [26] and the behaviour is similar to a parametric form of $\alpha_s(Q^2)$ expressed by Simon Capstick and N. Isgur in their study of baryons [9].

The single centre approximation for the size parameter defined in (19) for a p - quark wave function may be useful for the study of multiquark states like $qqqq$, $q\bar{q}q\bar{q}$ etc. Study of such states are beyond the scope of this paper. However, we might use it to obtain the average sizes of the baryons ($p = 3$). We find the average sizes $\langle r_o^2\rangle^{1/2}$ in $fermi$ as 0.78, 0.73, 0.69, 0.65, 0.47, 0.46, 0.45, 0.25 etc. for $N - \Delta$, $\Sigma - \Sigma^*$, $\Xi - \Xi^*$, Ω^-, $\Sigma_c - \Sigma_c^*$, $\Xi_c - \Xi_c^*$, Ω_c^0, $\Sigma_b - \Sigma_b^*$ etc. respectively. These results are comparable to the expected sizes of the baryons. There is wide scope for the applications of such simplified approximations in the study of multi-particle states.

In conclusion it suggests, the success of the suitable phenomenological parameterisation employed in this paper is responsible, in our opinion, for the satisfactory results obtained for the predictions of the masses of the open flavour hadrons and the leptonic decay widths of the vector mesons. The extension of the present study for the various properties of hadrons vis-a-vis radiative transition rates, other hadronic decays and the negative parity states of baryons, hybrids and glueballs are in progress.

This work is being supported by Department of Science and Technology, Govt. of India through a major SERC project.One of the authors (PCV) would like to thank the Director, Institute for Plasma Research, Bhat, Ghandhinagar for extending hospitality during a summer visit at the initial stages of this work and the Director, IUCAA, for the excellent library facilities made available. We would also like to thank the referee for his valuable suggestions.

References

1. CLEO Collaboration, Chen, W. Y. et al., Phys. Lett. **B 243** (1990) 169 CLEO Collaboration, Avery, P. et al., Phys. Rev. Lett. **75** (1995) 4364 CLEO Collaboration, Gibbons, L. et al., Phys. Rev. Lett. **77** (1996) 810
2. DELPHI Collaboration, DELPHI Report No. 95 - 107 (unpublished)
3. Particle Data Group, Phys. Rev. **D 54**, (1996) 25
4. Iachello,F., Mukhopadhyay, N. C. and Zhang L., "Spectrum generating algebra for string like mesons : mass formula for $q\bar{q}$ mesons" Phys. Rev. **D 44** (1991) 898
5. Quigg, C. and Rosner, J., Phys. Reports, **56**, 167, (1979) 222
6. Buchmüller and Tye, Phys. Rev. **D 24** (1981) 132
7. Isgur N., Karl, G., Phys. Rev. **D 18** (1997) 4187
8. Mitra, A. N. and Santhanam, I., Z. Physics C. Particles and Fields **8**, (1981) 33
9. Simon Capstick and N. Isgur, Phys. Rev. **D 34** (1986) 2809
10. Godfrey and Isgur, Phys. Rev. **D 32** (1985) 189
11. Vinodkumar, P. C. and Khadkikar, S. B., Phys. Lett. **B 329** (1994) 81 and Errata **340**, (1994) 264
12. Khadkikar, S. B. and Gupta, S. K., Phys. Lett. **B 124**, (1983) 523
13. Khadkikar, S. B. and Vinodkumar P. C., Pramana, J. Phys. **29**, (1987) 39
14. Vinodkumar, P. C., Vijayakumar, K. B. and Khadkikar, S. B., Pramana, J. Phys. **39**, (1992) 47
15. Khadkikar, S. B. and Vijayakumar, K. B., Phys. Lett. **B 254**, (1991) 320
16. Vijayakumar, K. B. and Khadkikar, S. B. Nuclear Physics **A 556**, (1993) 396
17. Vinodkumar, P. C., Bannur, V. M. and Khadkikar S. B.,"Hadronic masses in the open flavours from relativistic harmonic confinement model", International Nuclear Physics Conference, Beijing, China, Book of Abstracts (1995) 3-15
18. Gottfried K. and Weisskopf V. F., "Concepts of Particle Physics - Vol. II" Oxford University Press, New York (1986) p. 397-98
19. Attilio Cucchieri and Daniel Zwanziger, Phys. Rev. Lett. **78** (1997) 3814
20. Moshinsky, M., "The Harmonic Oscillator in Modern Physics : From Atoms to Quarks", Gordon and Breach Science Publishers, New York, Chapter 2 (1969)
21. Chen-Yu-Qi and Oakes Robert J., Phys. Rev. **D 53**, (1996) 5051
22. Close, F. E., "An introduction to quarks & partons", Academic Press, Sec. 17.4.1, (1979) 399
23. Giammarco, J., Franklin, J., Nucl. Phys. **A 585** (1995) 450
24. Bannur, V. M., Ph.D. Thesis, "Studies in covariant soliton dynamics", B.C.C.N. T86/121/161 (1988)
25. Abe, F. et al, Phys. Rev. Lett. **74** (1995) 2626 and Abachi, S. et al *ibid* (1995) 2632
26. Varun Sheel (1996) Ph. D. Thesis, Gujarat University, "Vacuum structure in QCD and hadronic correlators"